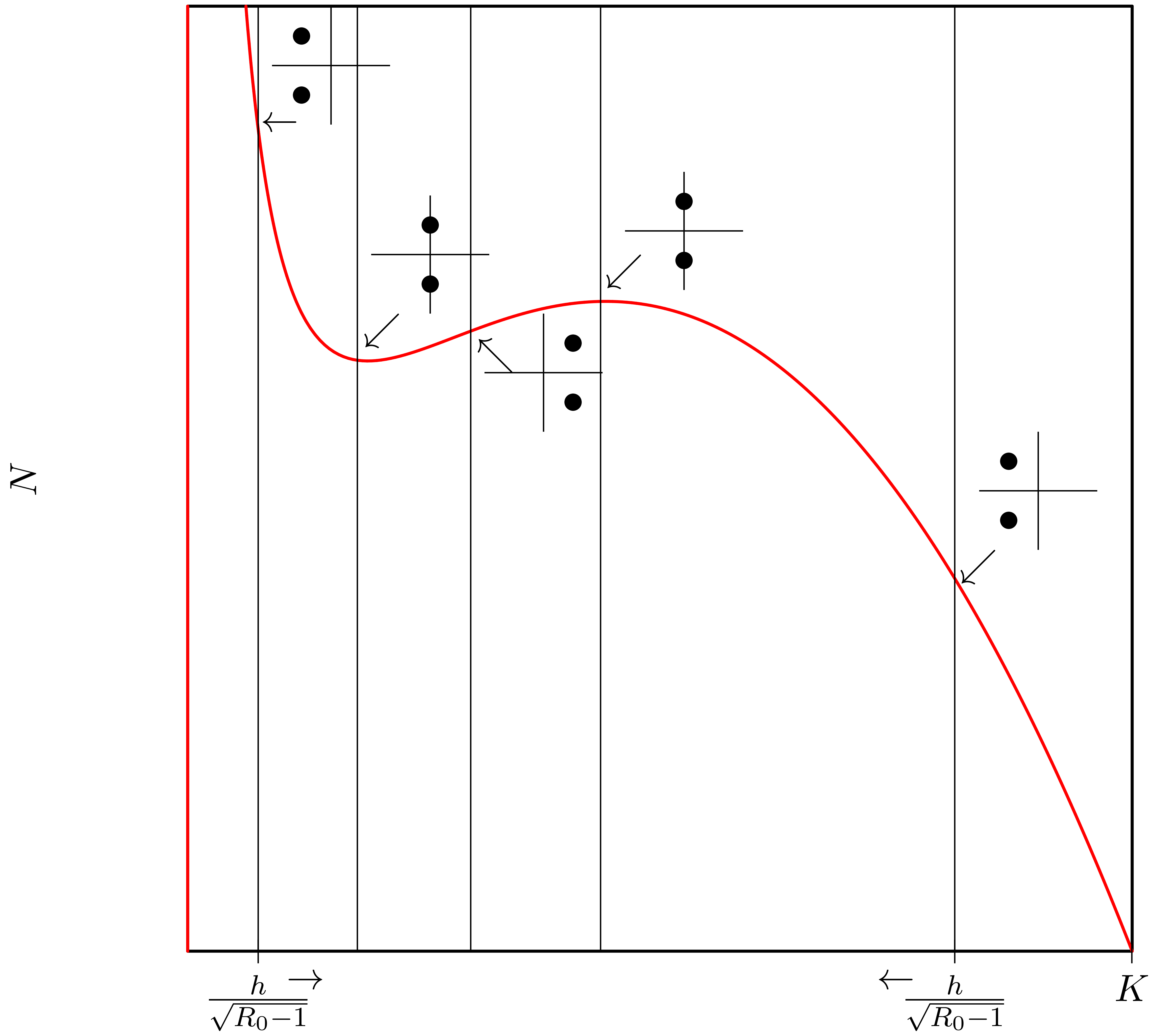
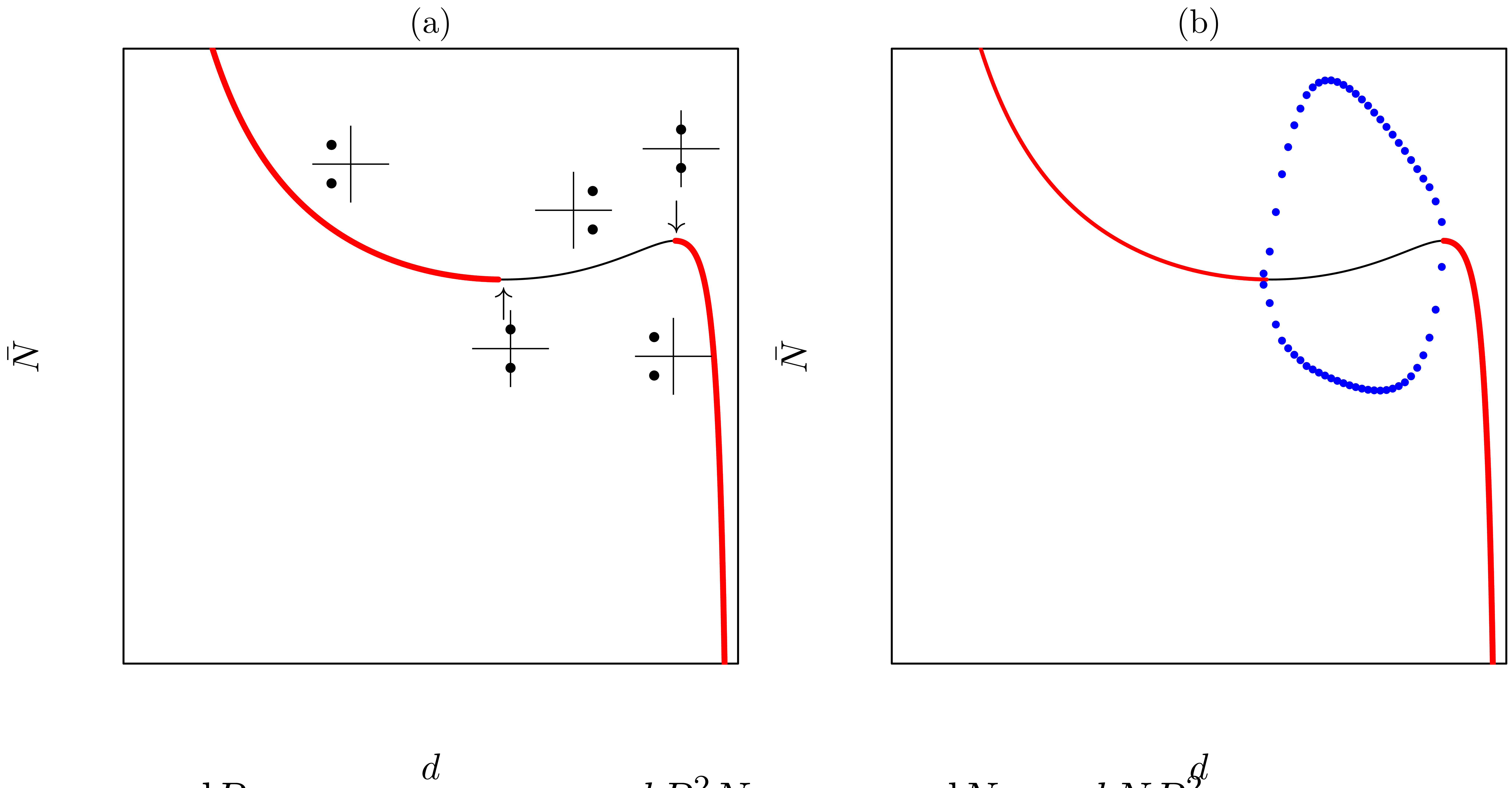
Chapter 11: Bifurcation analysis



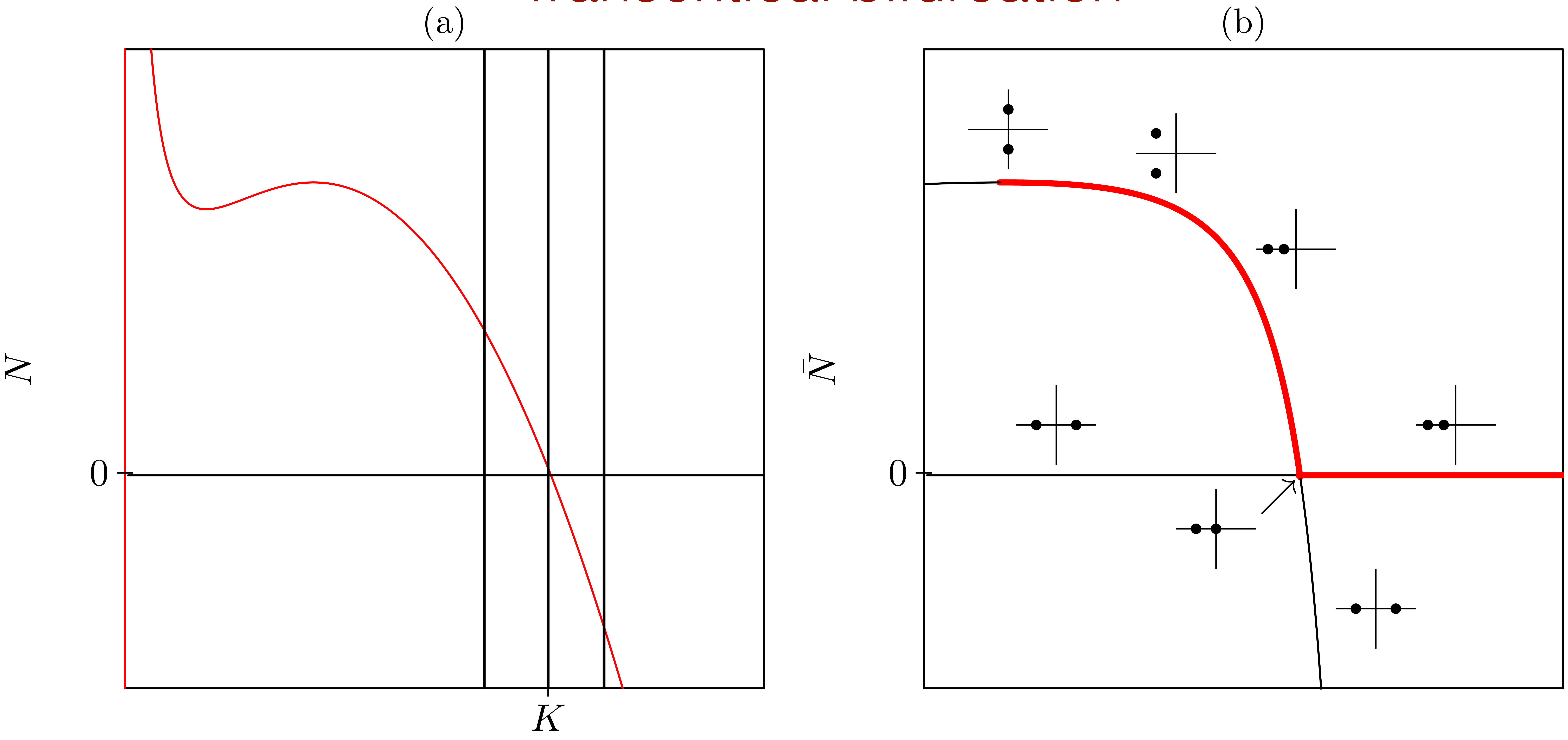
$$\frac{\mathrm{d}R}{\mathrm{d}t} = rR(1 - R/K) - \frac{bR^2N}{h^2+R^2}R \quad \text{and} \quad \frac{\mathrm{d}N}{\mathrm{d}t} = \frac{cbNR^2}{h^2+R^2} - dN \ ,$$

Hopf bifurcation



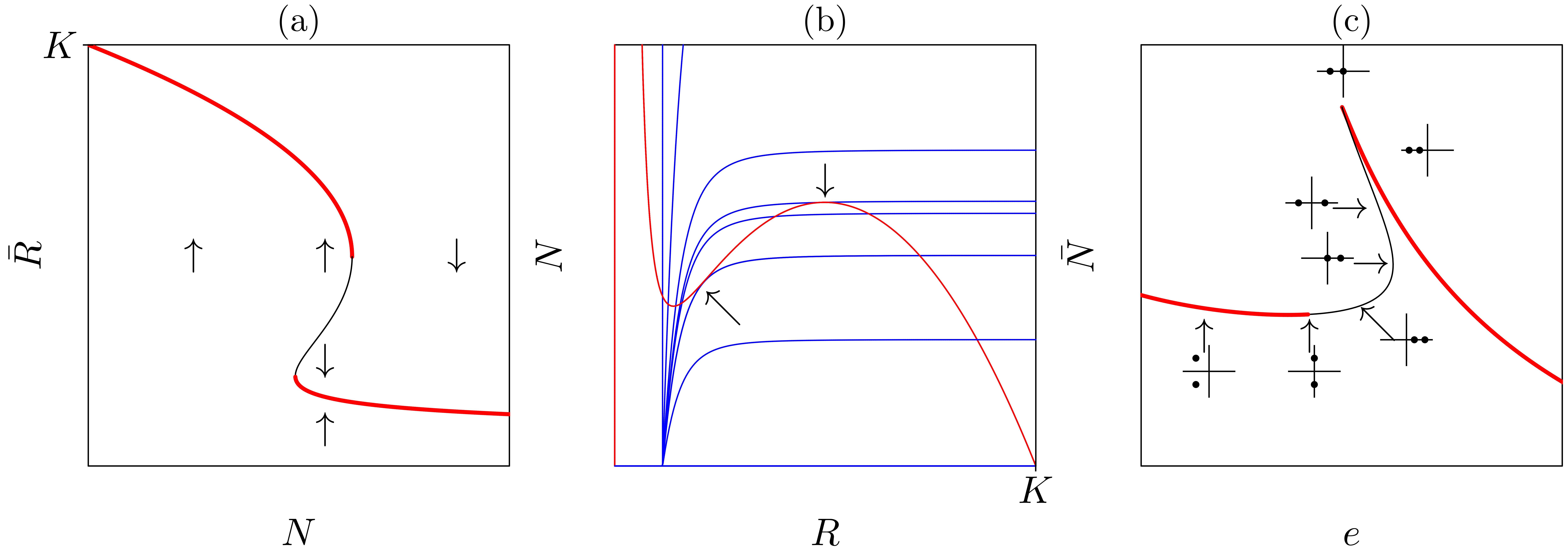
$$\frac{\mathrm{d}R}{\mathrm{d}t} = rR(1 - R/K) - \frac{bR^2N}{h^2 + R^2} \quad \text{and} \quad \frac{\mathrm{d}N}{\mathrm{d}t} = \frac{cbNR^2}{h^2 + R^2} - dN \;,$$

Transcritical bifurcation



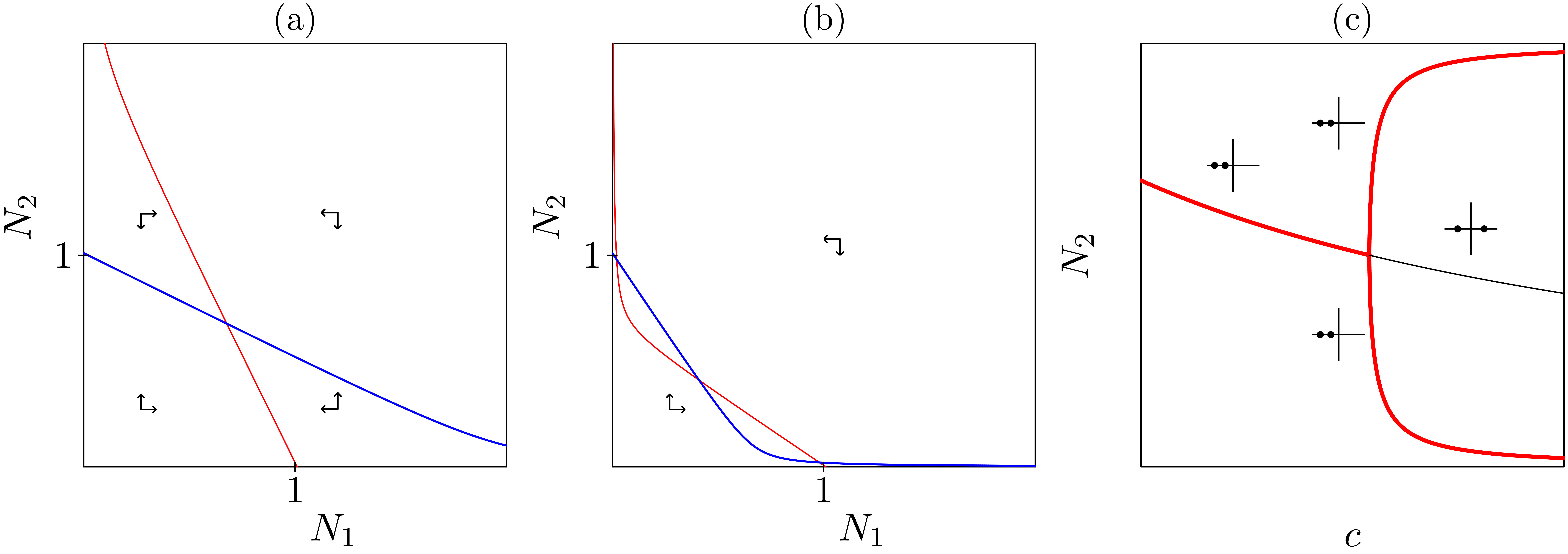
$$\frac{\mathrm{d}R}{\mathrm{d}t} = rR(1 - R/K) - \frac{bR^2N}{h^2 + R^2} \quad \text{and} \quad \frac{\mathrm{d}N}{\mathrm{d}t} = \frac{cbNR^2d}{h^2 + R^2} - dN \ ,$$

Saddle node bifurcation



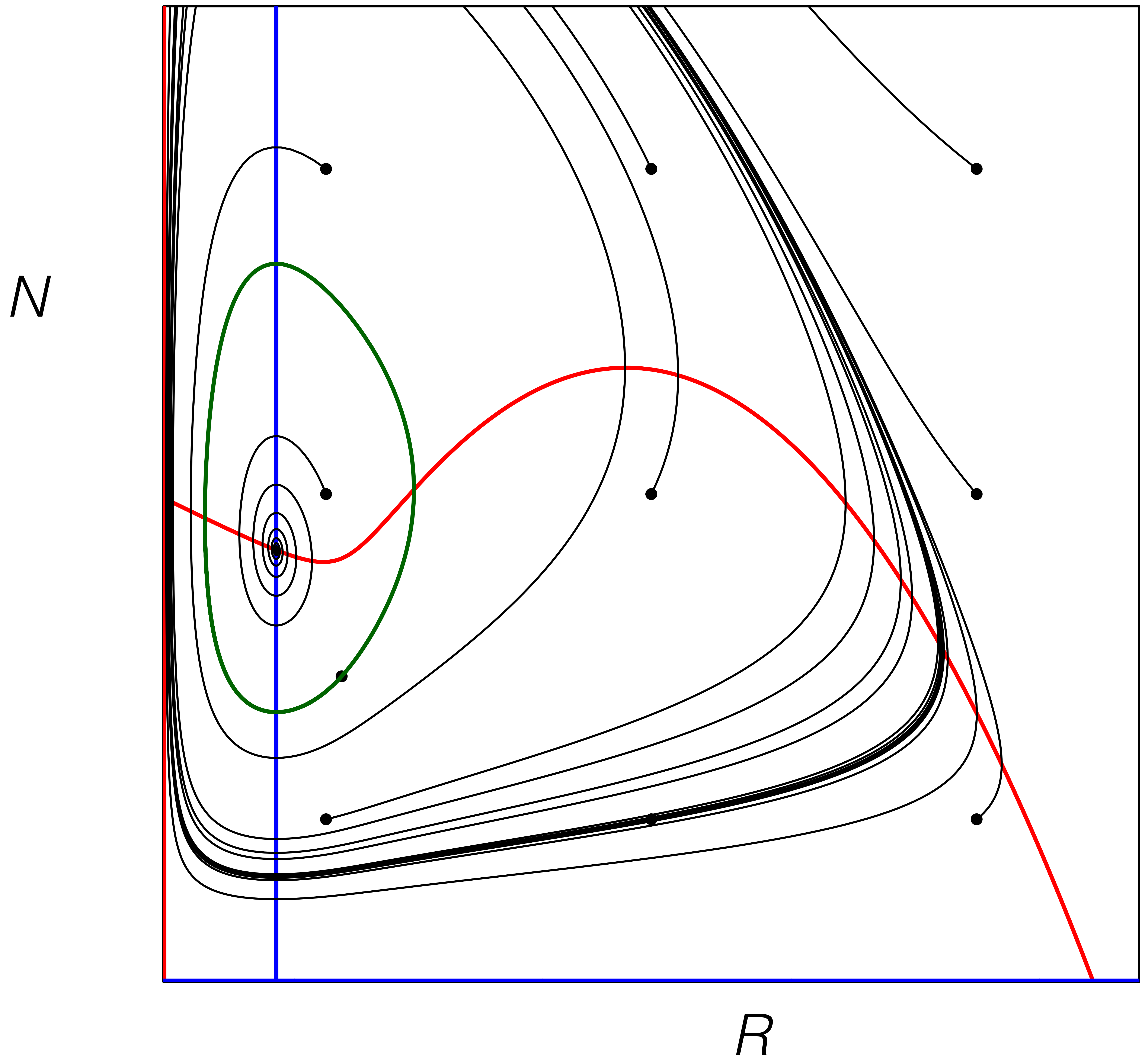
$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{cbNR^2}{h^2 + R^2} - dN - eN^2$$

Pitchfork bifurcation



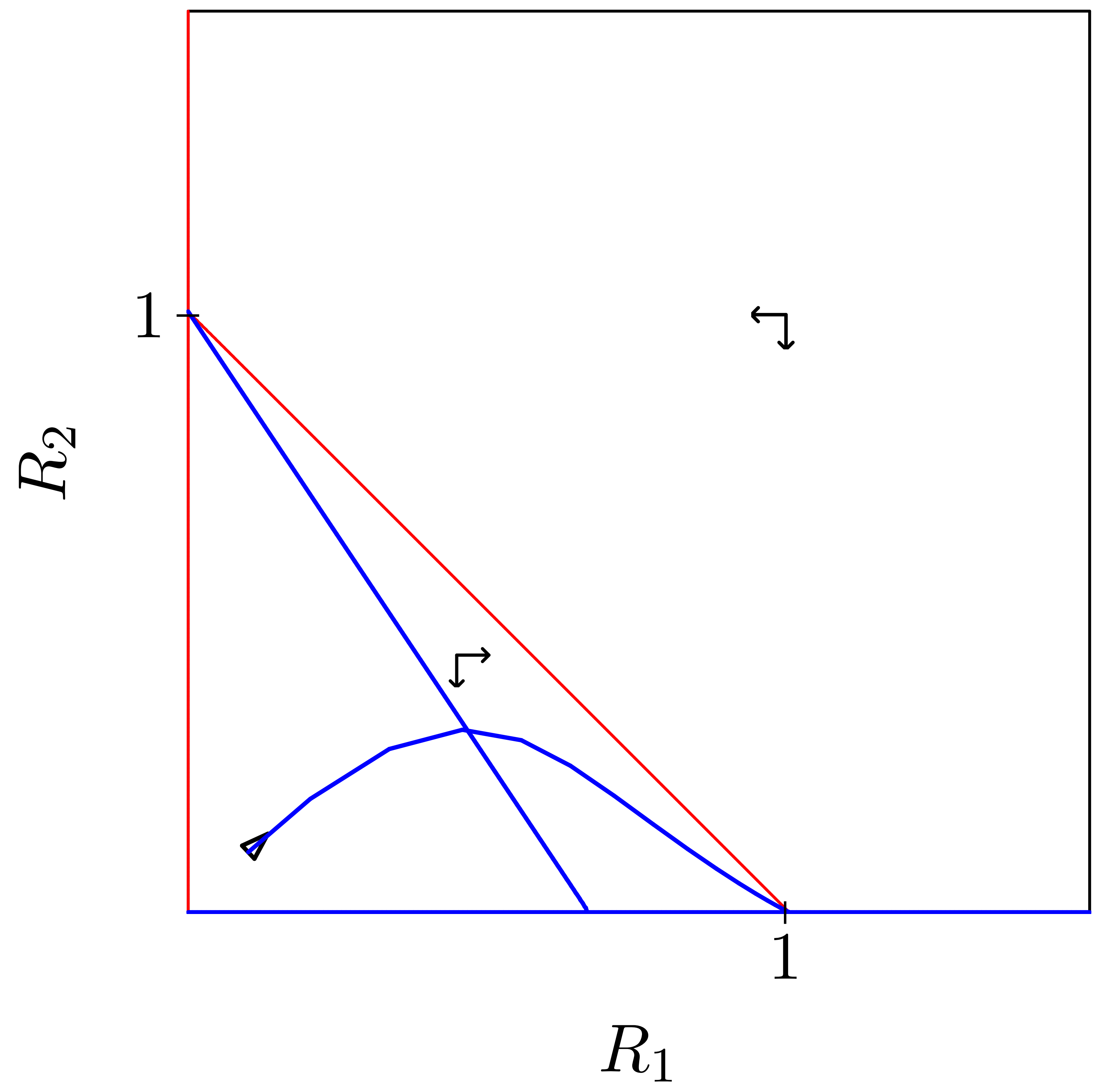
$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = i + rN_1(1 - N_1 - cN_2) \quad \text{and} \quad \frac{\mathrm{d}N_2}{\mathrm{d}t} = i + rN_2(1 - N_2 - cN_1)\ .$$

Unstable limit cycle



Chaos in a one consumer two resources model

$$\frac{\mathrm{d}R_1}{\mathrm{d}t} = R_1(1 - R_1 - \alpha_{12}R_2) - a_1 R_1 N \ ,$$
$$\frac{\mathrm{d}R_2}{\mathrm{d}t} = R_2(1 - R_2 - \alpha_{21}R_1) - a_2 R_2 N \ ,$$
$$\frac{\mathrm{d}N}{\mathrm{d}t} = N(ca_1R_1 + ca_2R_2 - 1) \ ,$$



a_{12}=1, a_{21}=1.5

a_2=1, c=0.5:
predator needs R_1

Vary a_1

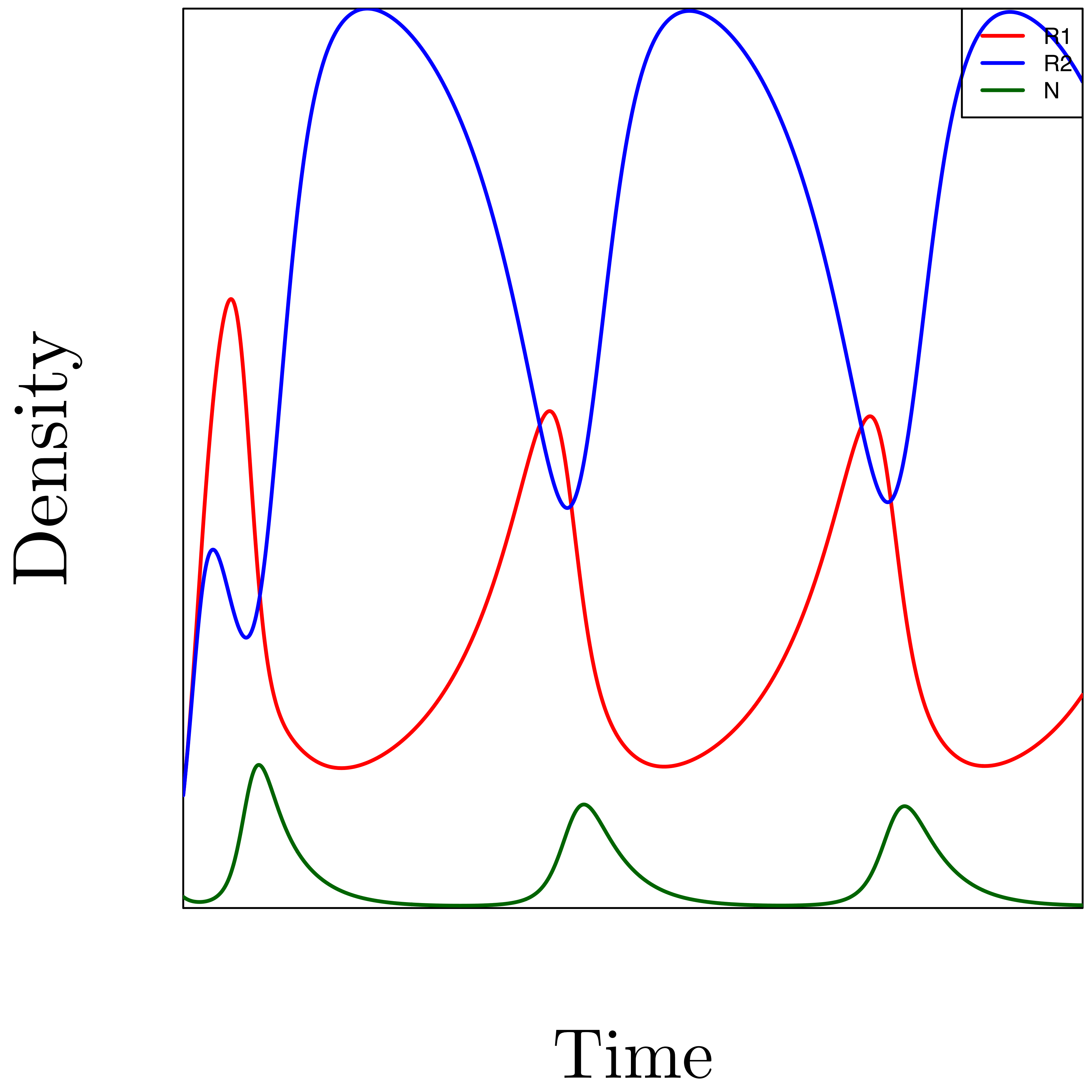
Chaos in a one consumer two resources model

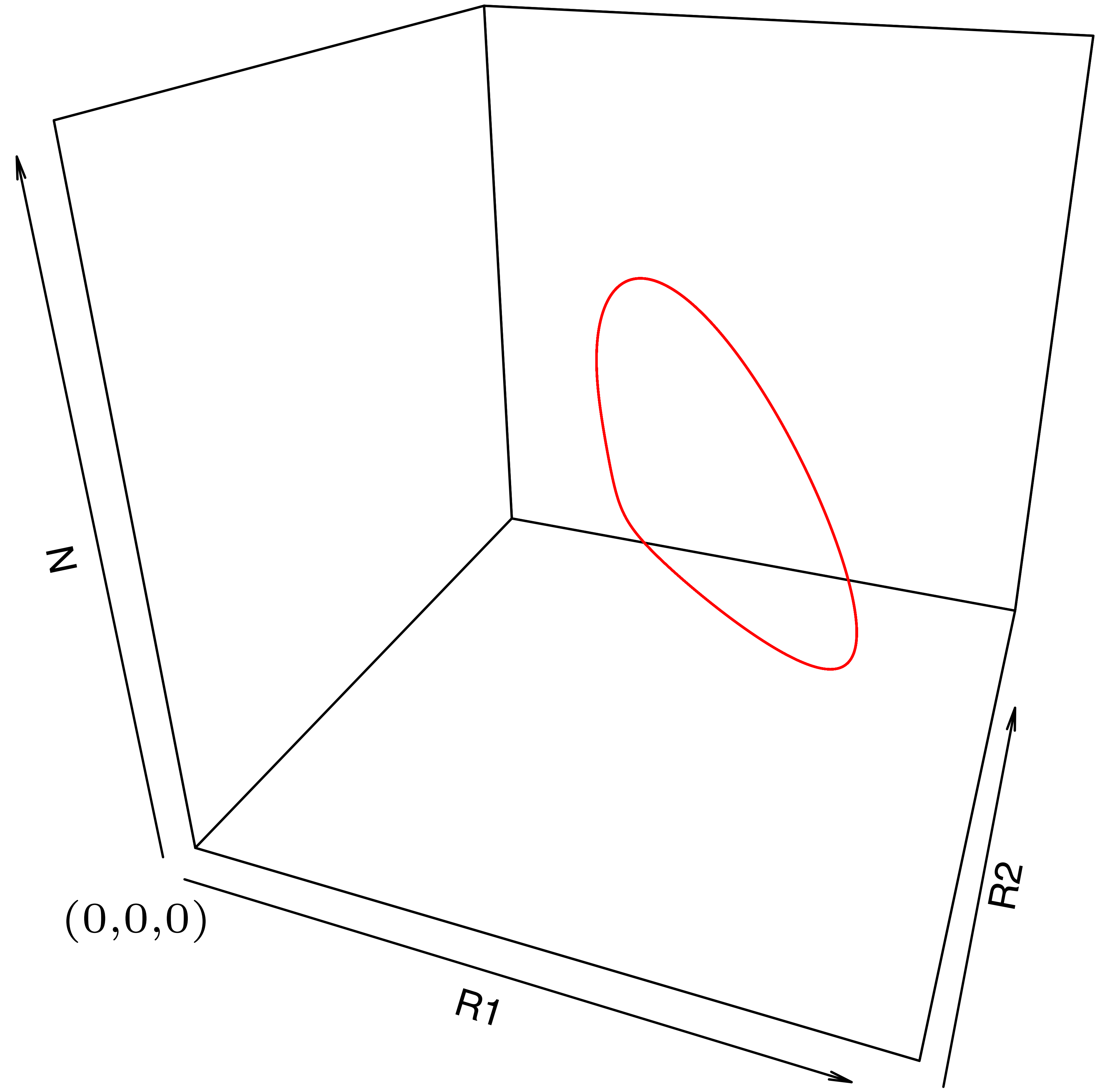
$$\frac{\mathrm{d}R_1}{\mathrm{d}t} = R_1(1 - R_1 - \alpha_{12}R_2) - a_1 R_1 N \ ,$$
$$\frac{\mathrm{d}R_2}{\mathrm{d}t} = R_2(1 - R_2 - \alpha_{21}R_1) - a_2 R_2 N \ ,$$
$$\frac{\mathrm{d}N}{\mathrm{d}t} = N(ca_1R_1 + ca_2R_2 - 1) \ ,$$

Hopf at a_1=5.5

a_1=6

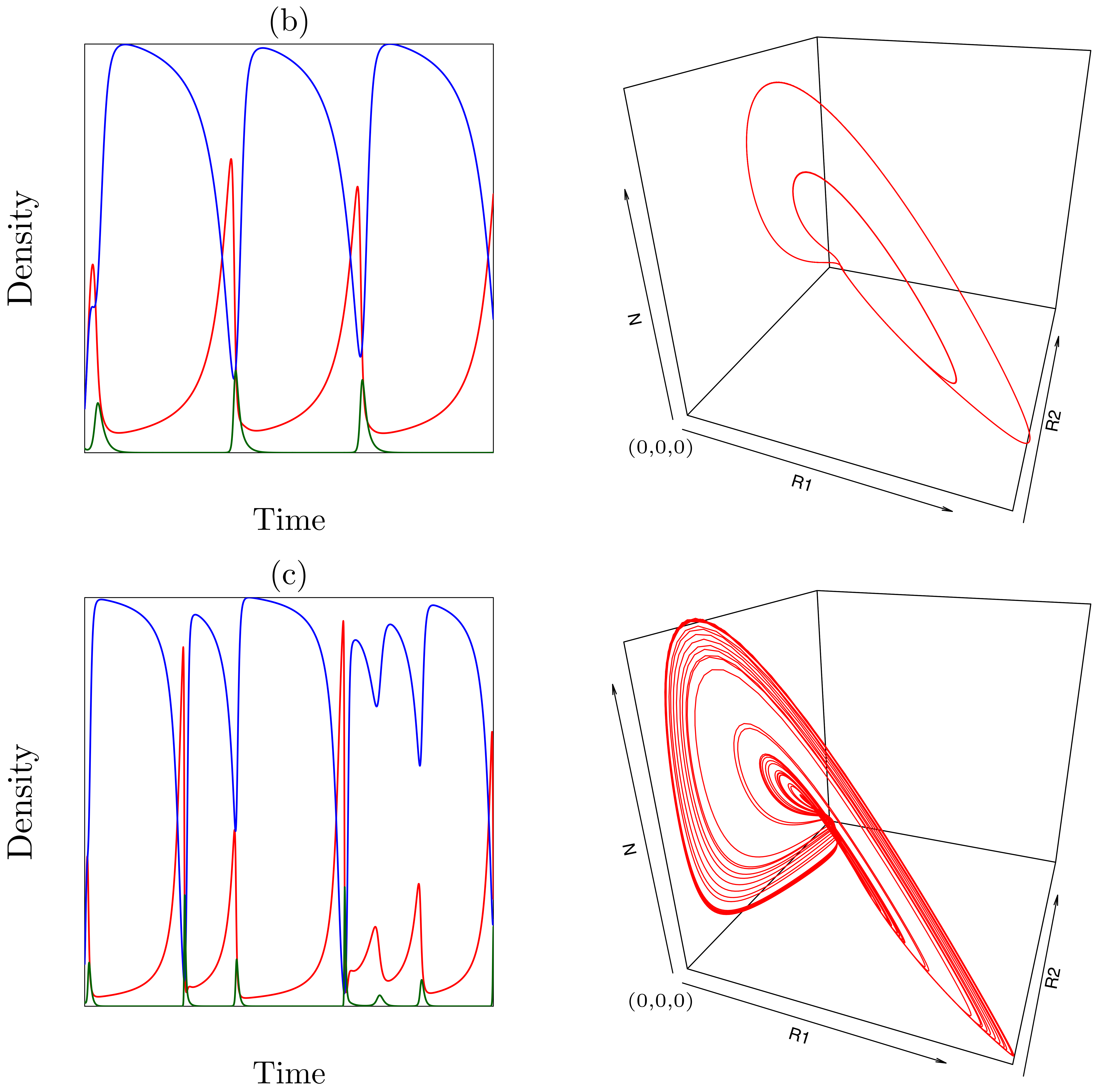
(a)





$a_1=8$

$a_1=10$



Period doubling cascade